A THEORETICAL AND EXPERIMENTAL INVESTIGATION OF THE PARAMETERS FOR AN ELECTRIC ARC

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A new method is presented for the determination of the temperature distribution over the cross section of a cylindrical arc, as well as of the volt-ampere characteristics, and these are compared with experimental data.

The volt-ampere characteristic represents an important operational indicator for gas-electric devices. A number of works [1–4] have been devoted to the calculation of electric-arcvolt-ampere characteristics. For arcs with approximately cylindrical symmetry these characteristics are frequently calculated according to formulas derived by Maecker [5] through solution of the differential energy-balance equation for an arc, with consideration of heat transfer by heat conduction. This equation is written in the form

$$\frac{1}{r}\frac{d}{dr}\left(r\lambda(T)\frac{dT}{dr}\right)+\sigma(T)E^2=0. \qquad (1)$$

The author of [5] linearizes the nonlinear differential energy-balance equation (1) for an arc by introducing the heat-conduction function $S=\int_{T_0}^{T}\lambda\,dT$, subsequently carrying out the linear approximation of the function $\sigma(S)$ according to the equal-area law. The approximating straight line is written in the form $\sigma = a(S - S_*)$.

After the transformations Eq. (1) assumes the form

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dS_1}{dr}\right)+aS_1E^2=0, \qquad (2)$$

where $S_1 = S - S_*$.

Having arbitrarily selected the quantity $\sigma_0(S_0)$ on the curve of the function $\sigma(S)$, we obtain specific values of the coefficients a and S_*. The author regards the value of σ_0 as having been taken on the discharge axis and uses it as a boundary condition in the solution of Eq. (2). For each σ_0 there will be specific values of the current I and the electric-field strength E. To obtain the volt-ampere characteristic it is necessary to carry out several approximations, making the calculation more complex. In this connection, it is important to develop a method for determining these parameters of the arc with a single linear approximation of σ from S, with an accuracy suitable for practical applications.

Unlike the Maecker method according to which the temperature distribution in the arc is determined from its axis to the periphery, we will calculate this distribution in the opposite direction. This is necessary to avoid having to specify the temperature on the axis of the arc and to derive simpler formulas for the determination of its parameters. Moreover, this method may be useful for the researchers engaged in measuring the temperatures in the peripheral zones of the arc and desirous of employing these measurements for a theoretical determination of the temperature within the central zones of the arc and on its axis.

In the peripheral zone of the arc, where no heat liberation takes place, we take the energy equation in its simplest form

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dS}{dr}\right)=0 \qquad (3)$$

with the boundary conditions

$$S(R)=0,\ S(r_1)=S_*. \qquad (4)$$

A solution for Eq. (3) with boundary conditions (4) is

$$S(r)=S_*\frac{\ln\frac{r}{R}}{\ln\frac{r_1}{R}}. \qquad (5)$$

For the boundary conditions of Eq. (2) we take the following expressions:

$$S_1(r_1)=0,\ S_{1r}(r_1)=S_*\frac{1}{r_1\ln\frac{r_1}{R}}. \qquad (6)$$

The right-hand member of the last expression represents the magnitude of the derivative function (5) when $r = r_1$.

When E = const, we have the Bessel equation (2); therefore, one of its solutions—having physical sense and satisfying boundary conditions (6)—will be the function

$$S_1(r)=S_*\frac{J_0\left(\frac{p}{r_1}r\right)}{pJ_1(p)\ln\frac{R}{r_1}}. \qquad (7)$$

The quantity E in this method is associated with r_1 by the expression

$$aE^2-\frac{p^2}{r_1^2}=0, \qquad (8)$$

which is derived from (2) after substitution into the latter of function (7). We calculate the electric current with the formula

$$I=2\pi aE\int_0^{r_1}S_1(r)\,rdr. \qquad (9)$$

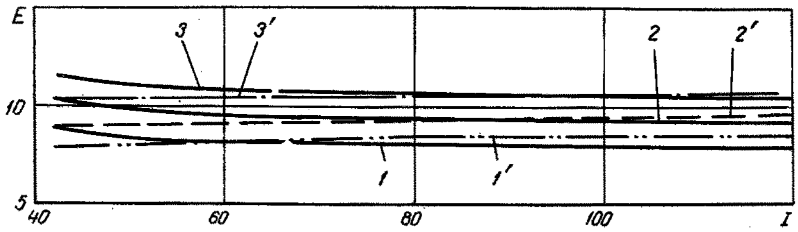

Fig. 1. Change in temperature radius of arc in argon (I = 60 A): 1) theoretical curve 2R = 0.6 cm (E = 6.7 V/cm; 1') experimental curve 2R = 0.6 cm (E = 6.7 V/cm); 2) theoretical curve, 2R = 0.5 cm (E = 9 V/cm); 2') experimental curve, 2R = 0.5 cm (E = 8.1 V/cm); 3) theoretical curve, 2R = 0.4 cm (E = 12 V/cm); 3') experimental curve, 2R = 0.4 cm (E = 11.2 V/cm).

It should be noted that the radius r_1 of the current-conducting zone of the arc is not determined experimentally, but selected arbitrarily from the interval $0 < r_1 < R$. Here a given specific value of r_1 from Eqs. (8) and (9) is used to calculate I and E which are used to construct the volt-ampere characteristic. This last is compared with the volt-ampere characteristic derived experimentally.

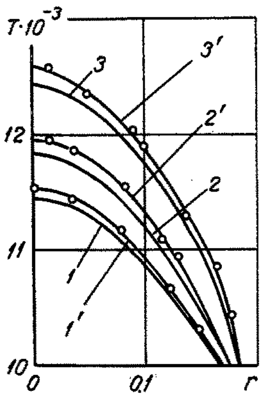

Fig. 2. Volt-current characteristic of an arc: 1 and 1') theoretical and experimental curves for 2R = = 0.6 cm; 2 and 2') theoretical and experimental curves for 2R = = 0.5 cm; 3 and 3') theoretical and experimental curves for 2R = = 0.4 cm.

However, the radius r_1 can be eliminated from Eqs. (8) and (9), so that for I we obtain the following expression:

$$I = 2\pi \frac{S_*}{E \ln \frac{RE\sqrt{a}}{p}}. \quad (10)$$

The value of E in this case is regarded as given, with r_1 being the sought quantity. To determine S(r) from (5) and (7) we must know the magnitude of r_1 which is determined from the value of E in formula (8).

We find the temperature T(r) on the basis of S(r) from the S(T) curve which is determined by the integration of $\lambda(T)$ with respect to T within limits from T_0 to T.

Let us examine several variants for the determination of the coefficients a and S_* and let us ascertain the feasibility of using these with the given method.

1. If the coefficients a and S_* are determined by the equal-area law [5], then in a calculation—according to the proposed method—the quantity σ_0 must be located on the discharge axis. This is possible only after selection of several values of $2r_1$ (or several values of E, with I defined according to Eq. (10)). In this case we obtain only a single point on the volt-ampere characteristic. We note that this problem is analogous to the Maecker problem, but carried out in the opposite direction. We have to repeat the Maecker problem in reverse sequence for better clarification of the distinction between the proposed method and that of Maecker, as well as to demonstrate that this difference is meaningful for the further development of the method to calculate the arc parameters.

2. If the coefficients a and S_* are defined according to the law of least squares [6, 7], we are not bound in the calculation based on the proposed method to specify σ_0 on the discharge axis. For each value of the diameter $2r_1$ (or for one value of E) we therefore obtain a uniquely defined value for the current I. Schmietz [7] solved the problem earlier with this approximation, but he, as did Maecker, did the calculation from the axis of the arc to its periphery.

3. In addition to the above-cited methods of linearizing the nonlinear energy-balance for the arc, we frequently employ numerical methods for its solution [4]. The differential equation is transformed into an integral equation and is solved by the method of successive approximations, with given radius of the arc and gas temperature on the axis. Among the shortcomings of this method is the fact that theoretical formulas cannot be derived, and the actual solution is presented in the form of curves. This is a cumbersome method and not considered here for that reason.

The indicated methods of determining the coefficients a and S_* are laborious and these coefficients are therefore most simply found graphically, assuming the accurary needed for the calculations, not exceeding 10%.

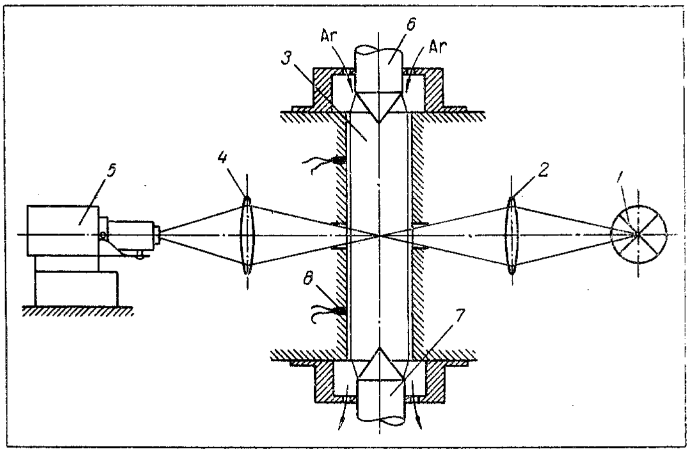

Fig. 3. Circuit for measuring arc temperature: 1) filament lamp SI-6-100; 2) lens; 3) arc; 4) lens; 5) spectrograph ISP-5; 6) cathode; 7) anode; 8) thermocouple.

The arc parameters were calculated in accordance with the proposed method. The results of the calculation are given in Figs. 1 and 2 (with experimental data shown here as well), and we can see from these that the divergence in the results is very small. This last fact is significant and confirms the feasibility of doing the calculations by the simpler method.

The experiments were carried out on an installation whose basic design is shown in Fig. 3. The arc temperature was measured from additions of hydrogen (the H_β line) and from the absolute intensities of the 4300 angstrom unit Ar line. The graphs show the diameters and combustion regimes for the arcs, with a gas flow rate not exceeding 0.001 g/sec.

The theoretical section was developed by V. E. Ionin; the experiments were carried out by N. G. Shesterkin and V. S. Popenko.

NOTATION

$\lambda(T)$ is the thermal conductivity; $\sigma(T)$ is the electrical conductivity; r is the instantaneous radius; R is the channel radius; r_1 is the radius of current-conducting channel; E is the intensity of the electric field; S_* is the value of the heat-conduction function at $T = T_*$; T_0 is the wall temperature of a channel having radius R; T_* is the value of the temperature for $\sigma \cong 0$; a is the proportionality factor for the linear function $\sigma(S)$; $p = 2.403$ is the least root of the equation $J_0(x) = 0$, $J_1(p) = 0.521$.

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